Relation between luminous efficiency function and color matching functions

Hirohisa YAGUCHI*

Abstract

Color matching functions and luminous efficiency functions by flicker photometry for a 2° foveal field, were measured for two observers at a retinal illuminance of 100 trolands. The synthetic luminous efficiency function determined as a linear combination of color matching functions was compared with the measured one for a given observer. Taking into account the uncertainties of measurements, the synthetic functions were in good agreement with the measured one.

1. Introduction

Analyzing the relationships among various visual functions; color matching functions, luminous efficiency function, chromatic valence functions, wavelength discrimination, foveal threshold, and so on, gives us a great deal of knowledge of how the visual system processes color information. However, inter-observer deviation of visual functions caused by individual differences of macular pigment and lens absorption is an obstacle in analyzing the data. To make a quantitative model of color vision or to test an existing color vision model, therefore, it is appropriate to measure a set of visual functions for a given observer and apply it to the models. Among various visual functions, color matching functions and luminous efficiency function are the most important ones, because they provide not only the basis of color vision models but also the basis of colorimetry and photometry.

The 1931 CIE Standard Observer color matching functions were derived from Wright's and Guild's colorimetric data (not color matching functions but chromaticity coordinates of spectral colors) and the 1924 CIE $V(\lambda)$ function. This derivation relies on the assumption that the $V(\lambda)$ function is a linear combination of color matching functions. There have been several studies that examine the validity of this important assumption.

The first study was made by Stiles.[1] He measured 2° color matching functions for 10 observers. He also determined the luminous efficiency function for a 2° field in the blue end of spectrum by heterochromatic brightness matching for 28 observers including the 10 observers for whom color matching functions had also been measured. A linear combination of the mean color matching functions using the observer's own direct measured luminous efficiencies at the primary colors, were compared with the mean of his direct measured luminous efficiency function. The synthetic luminous efficiency function was generally in good agreement with the directly measured one, but the values were relatively low in the blue region. Stiles suggested that a similar test should be applied to the complete luminous efficiency function determined for the same observers by some acceptable procedure of heterochromatic photometry, such as flicker photometry for example.

Later on, Sperling[2] carried out a similar test with six observers. He measured the luminous efficiency functions using two methods, heterochromatic brightness matching and flicker photometry. He compared each of the luminous efficiency functions with a linear combination of their average color matching functions weighted by the respective luminous efficiencies at the primaries. Sperling found that deviations of the synthetic luminous efficiency function from the real one were large through part of the spectrum for brightness matching but small for flicker photometry. His results imply that brightness matching does not obey the additivity law, whereas the flicker photometry does. Al-

* Department of Image Science and Engineering
Faculty of Engineering, Chiba University
1-33 Yayoi-cho, Chiba, 260 Japan

though Sperling concluded that the deviations found using the flicker photometry did not allow rejection of the linearity hypothesis between color matching functions and the luminous efficiency function, Estevez[3)] pointed out that the differences appeared to be systematic and they were similar to Stiles's results.

Richards and Luria[4)] also measured color matching functions and luminous efficiency functions by flicker photometry for three observers at three luminance levels in the mesopic region. They found no significant difference between the synthetic luminous efficiency function and the measured one at all luminance levels.

In the present study, we re-examined this relationship using the same observers.

2. Methods and results

2.1 Color matching functions

The color matching experiments were done on the NRC Trichromator originally designed by Stiles. A schematic view and details of the Trichromator are described elsewhere.[5),6)] Color matching functions were obtained by the maximum saturation method. A 2° bipartite field was displayed in a dark surround with Maxwellian view. The upper half of the bipartite field provided two of the primary stimuli of the Trichromator at wavelength λ_R = 645.2 nm (15,500 cm^{-1} in wavenumber), λ_G = 526.3 nm (19,000 cm^{-1}), and λ_B = 444.4 nm (22,500 cm^{-1}). The lower half provided the monochromatic test stimuli in the range 408 nm to 690 nm in 250-cm^{-1} wavenumber steps, and a desaturating primary. The observer controlled the radiance of three primary stimuli to make a color match between the upper half and the lower half of the field by the method of adjustment. The retinal illuminances of the test stimuli between 417 nm and 690 nm were 100 Td. In the shorter wavelength region however, they were 75 Td for 412 nm and 35 Td for 408 nm because of insufficient light.

The test spectrum was divided into two sets, one was from 408 nm to 690 nm in 500-cm^{-1} wavenumber steps, and the other from 412 nm to 678 nm in 500-cm^{-1} steps. Each set was run through in one of two directions, that is, from blue to red or from red to blue, in a separate experimental session. One match was made at each test wavelength. It took about one hour to complete one session. Two observers; HY (34 years) and ZF (40 years), joined this experiments. Four experimental sessions were repeated for each observer.

Color matching functions were determined directly from the radiant powers of a test stimulus and three primaries. If radiant powers $R(\lambda)$, $G(\lambda)$, and $B(\lambda)$ of the three primaries are required to match a test monochromatic stimulus (wavelength, λ) of radiant power $L(\lambda)$, the color matching functions $\bar{r}(\lambda)$, $\bar{g}(\lambda)$, and $\bar{b}(\lambda)$ are determined as follows,

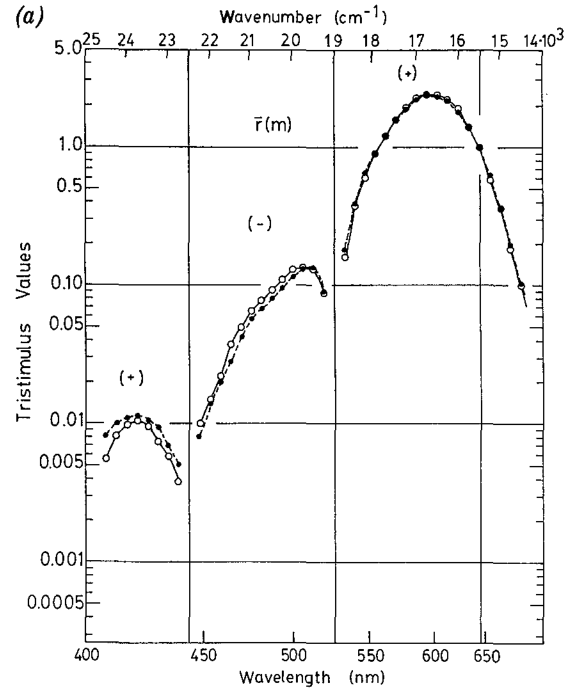
$$\begin{aligned} \bar{r}(\lambda) &= R(\lambda)/L(\lambda) \\ \bar{g}(\lambda) &= G(\lambda)/L(\lambda) \quad \cdots\cdots\cdots\cdots\cdots \quad (1) \\ \bar{b}(\lambda) &= B(\lambda)/L(\lambda)\,. \end{aligned}$$

The amount of the desaturating primary stimulus is given a negative sign.

The raw data of the color matching functions from the four sessions were plotted on a logarithmic scale, and then smoothed and normalized so that the color matching function was unity at the wavelength of the respective primary. The resultant color matching functions for two observers are shown in **Table 1** and **Fig. 1**.

2.2 Luminous efficiency function by flicker photometry

The flicker photometry experiment was also done on the same Trichromator. In this experiment, a 2° full field was presented in a dark surround. A reference white stimulus was provided by mixing three primary stimuli of the Trichromator to give the chromaticity coordinates of D_{65} white (x = 0.313, y = 0.329) and a retinal illuminance of 100 Td. The test stimulus was one of the monochromatic stimuli, from 408 nm to 690 nm in 250-cm^{-1} wavenumber steps. The reference stimulus and the test stimulus were presented alternately. The flicker frequency was set at 20 Hz throughout the test wavelength region. The observer adjusted the radiance of the test stimulus to determine the minimum flicker point. In some wavelength regions, for example in the green-yellow region, flicker disappeared completely over a certain small range of test radiances. In this case, the observer determined the middle point of the flicker disappearance range. Three repeats were made at each test wavelength in each session.



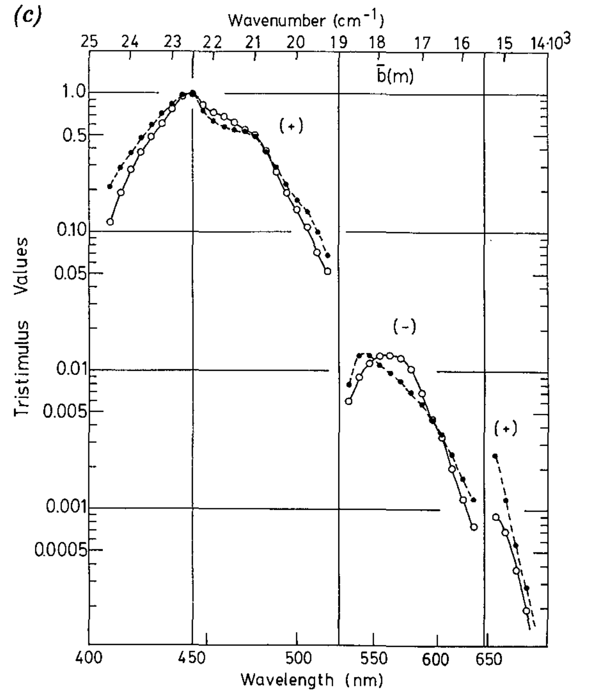
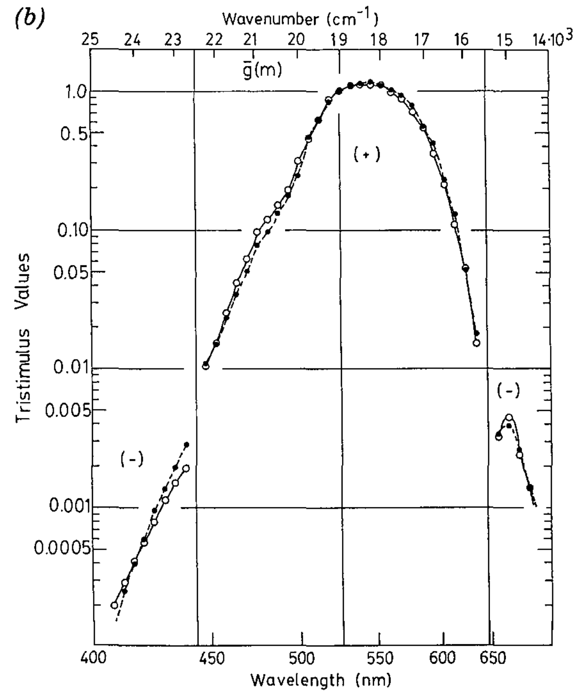


Fig. 1 *Color matching functions, open circles: observer HY, closed circles: observer ZF. (a): $\overline{r}(\lambda)$, (b): $\overline{g}(\lambda)$, and (c): $\overline{b}(\lambda)$.*



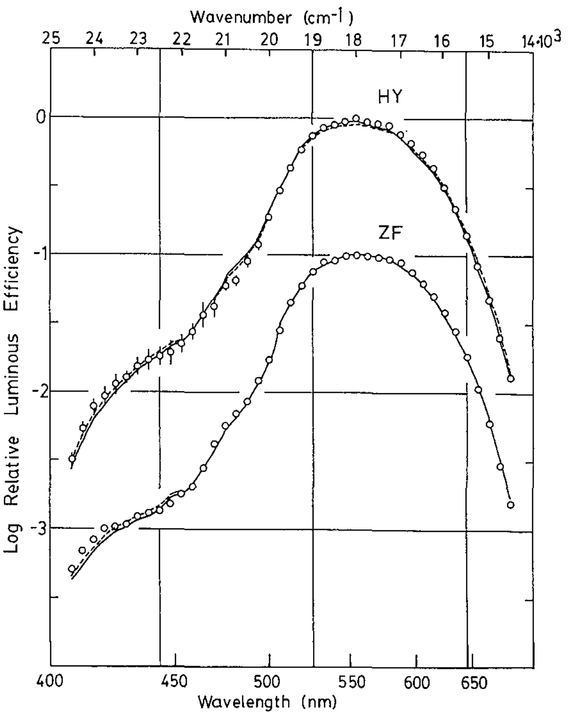


Fig. 2 *Luminous efficiency functions for two observers. ZF's ones are displaced vertically by one logarithmic unit. Open circles show the experimental data obtained by flicker photometry, solid curve shows a linear combination of color matching functions weighted by the luminous efficiencies at the respective primaries, and the dashed curve shows that weighted by coefficients minimizing the deviations from the experimental data.*

There were four sessions for one observer (HY), and two sessions for another (ZF). The luminous efficiency function was determined by the reciprocal of the energy of the test stimulus required to provide minimum flicker.

The luminous efficiency functions by flicker photometry for the same two observers are shown in Table 1. The values were normalized to unity at 555.6 nm. The luminous efficiency functions are plotted in **Fig. 2**. Vertical bars in HY's plots show ±1 standard deviation. Standard deviations were not calculated for ZF's results because of insufficient data.

Table 1. *Color matching functions by maximum saturation method and luminous efficiency functions by flicker photometry*

Subject	HY				ZF			
Wave-number m (cm-1)	$\bar{r}$(m)	Color matching functions $\bar{g}$(m)	$\bar{b}$(m)	Luminosity functions V(m)	$\bar{r}$(m)	Color matching functions $\bar{g}$(m)	$\bar{b}$(m)	Luminosity functions V(m)
14500	0.0982	-0.00137	0.00019	0.0131	0.104	-0.00140	0.00028	0.0158
14750	0.180	-0.00234	0.00037	0.0252	0.195	-0.00260	0.00056	0.0300
15000	0.360	-0.00440	0.00070	0.0483	0.360	-0.00390	0.00120	0.0601
15250	0.584	-0.00321	0.00090	0.0855	0.630	-0.00340	0.00250	0.107
15500	1.00	0.00000	0.00000	0.141	1.00	0.00000	0.00000	0.185
15750	1.38	0.0151	-0.00076	0.224	1.40	0.0180	-0.00120	0.282
16000	1.92	0.0532	-0.00120	0.320	1.80	0.0520	-0.00170	0.390
16250	2.23	0.111	-0.00199	0.443	2.20	0.130	-0.00250	0.512
16500	2.37	0.212	-0.00331	0.553	2.36	0.230	-0.00340	0.622
16750	2.39	0.349	-0.00448	0.671	2.40	0.420	-0.00440	0.757
17000	2.26	0.534	-0.00697	0.780	2.20	0.560	-0.00580	0.893
17250	1.93	0.715	-0.0105	0.889	1.90	0.790	-0.00700	0.948
17500	1.57	0.869	-0.0125	0.929	1.58	0.930	-0.00840	0.962
17750	1.21	0.990	-0.0129	0.942	1.20	1.03	-0.00980	0.979
18000	0.902	1.10	-0.0127	1.00	0.900	1.11	-0.0110	1.00
18250	0.608	1.13	-0.0113	0.955	0.660	1.17	-0.0128	0.991
18500	0.375	1.13	-0.00896	0.887	0.390	1.14	-0.0128	0.908
18750	0.161	1.11	-0.00598	0.845	0.180	1.09	-0.00800	0.895
19000	0.00000	1.00	0.00000	0.741	0.00000	1.00	0.00000	0.757
19250	-0.0771	0.864	0.0524	0.593	-0.0880	0.830	0.0680	0.608
19500	-0.130	0.625	0.0712	0.433	-0.135	0.620	0.100	0.440
19750	-0.134	0.443	0.107	0.294	-0.132	0.460	0.140	0.278
20000	-0.129	0.307	0.144	0.186	-0.115	0.240	0.170	0.171
20250	-0.110	0.193	0.191	0.118	-0.0960	0.175	0.220	0.120
20500	-0.0925	0.151	0.271	0.0900	-0.0800	0.130	0.290	0.0841
20750	-0.0774	0.118	0.384	0.0637	-0.0680	0.0960	0.380	0.0658
21000	-0.0652	0.0954	0.498	0.0590	-0.0570	0.0780	0.490	0.0558
21250	-0.0491	0.0618	0.548	0.0422	-0.0420	0.0500	0.530	0.0415
21500	-0.0374	0.0414	0.613	0.0363	-0.0280	0.0340	0.550	0.0274
21750	-0.0219	0.0253	0.688	0.0274	-0.0200	0.0230	0.580	0.0202
22000	-0.0147	0.0153	0.737	0.0218	-0.0139	0.0150	0.640	0.0177
22250	-0.00992	0.0104	0.837	0.0191	-0.00800	0.0110	0.760	0.0150
22500	0.00000	0.00000	1.00	0.0182	0.00000	0.00000	1.00	0.0136
22750	0.00378	-0.00189	0.944	0.0170	0.00500	-0.00280	0.970	0.0130
23000	0.00567	-0.00149	0.778	0.0150	0.00700	-0.00195	0.840	0.0122
23250	0.00737	-0.00113	0.618	0.0126	0.00940	-0.00135	0.720	0.0106
23500	0.00944	-0.00078	0.486	0.0112	0.0106	-0.00094	0.590	0.0102
23750	0.0105	-0.00056	0.375	0.00923	0.0113	-0.00060	0.480	0.00995
24000	0.00973	-0.00041	0.281	0.00773	0.0109	-0.00039	0.370	0.00822
24250	0.00813	-0.00030	0.189	0.00528	0.00990	-0.00025	0.290	0.00684
24500	0.00547	-0.00020	0.117	0.00314	0.00810	-0.00014	0.210	0.00519

3. Discussions

On the assumption that the heterochromatic additivity law holds for flicker photometry, the luminous efficiency function, $V_p(\lambda)$ should be predicted by a linear combination of color matching functions as follows:

$$V_p(\lambda) = l_R \bar{r}(\lambda) + l_G \bar{g}(\lambda) + l_B \bar{b}(\lambda), \qquad \cdots\cdots\cdots\cdots (2)$$

where coefficients l_R, l_G, and l_B are determined from the luminous efficiency values at the wavelengths of the respective primaries. These values were actually obtained by the flicker photometry. The coefficients are l_R = 0.141, l_G = 0.741, and l_B = 0.0182 for the observer HY, and l_R = 0.185, l_G = 0.757, and l_B = 0.0136 for ZF. The predicted luminous efficiency function for each observer is shown by a solid curve in Fig. 2. There are fairly good agreements between the measured and predicted curves (correlation coefficients were 0.9983 for the observer HY and

0.9990 for ZF), but the predicted curves are slightly lower than the measured ones (not exceeding 0.1 logarithmic units) in the violet wavelength region shorter than 444 nm. This is consistent with the data obtained by Stiles-Burch[1] and Sperling[2]. However, taking into account the variability in both luminous efficiency and color matching measurements, these differences in the short wavelength region are not significant. Furthermore, if we are simply interested in testing whether the luminosity function by flicker photometry is a linear combination of color matching functions, then the combination of color matching functions may be determined by providing the best fit to the measured luminosity function. The dashed curves in Fig. 2 represent the synthetic functions using the coefficients determined by minimizing the squared deviations of the logarithmic values of the synthetic luminosities from those of the measured ones for each observer. The coefficients are l_R = 0.1562, l_G = 0.698, and l_B = 0.0197 for the observer HY, and l_R = 0.1825, l_G = 0.7448, and l_B = 0.015 for ZF. These curves provide the best possible fit to the measured curves (correlation coefficients are 0.9991 for both observers).

From the view point of making a precise color vision model however, we can not completely neglect the differences observed in the short wavelength region. There are two possible explanations for these differences. One is photometric additivity failure, that is, additivity failure for the flicker photometry; and the other is colorimetric additivity failure, that is, additivity failure of color matching functions. The former failure can be rejected because the additivity for flicker photometry has been confirmed by many independent studies.[7] On the other hand, the colorimetric additivity failure has been observed by Crawford[8] and Wyszecki.[9] They carried out color matching experiments using two methods, the maximum saturation method and the Maxwell method. In the Maxwell method, one half field provided a constant white and the other half field comprised a test monochromatic stimulus and two of the three primary stimuli. Color matches were always made on the white field independent of the test wavelength. If the proportionality and additivity laws of color matching hold strictly, color matching functions using the Maxwell method should be identical to those using the maximum saturation method. However, Crawford's and Wyszecki's results showed small but systematic differences between two sets of color matching functions using both methods, particularly in the short wavelength region. The differences between the synthetic function and the measured one observed in the short wavelength region might be associated with this failure of additivity law of color matching, but further work is needed to resolve this difficult problem.

It is emphasized once again that a linear combination of color matching functions is in good agreement with the luminous efficiency function using flicker photometry. Assuming that the color matching functions are linearly related to the spectral sensitivities of the cone receptors, the present result implies that the output from the cone receptors is linearly transformed to the luminance or achromatic visual channel which is responsible for flicker photometry.

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